A Theory of Production

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A THEORY OF PRODUCTION[1]

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1. *Introduction.*—The progressive refinement during the recent years in the measurement of the volume of physical production in manufacturing suggests the possibility of attempting (1) to measure the changes in the amount of labor and capital which have been used to turn out this volume of goods, and (2) to determine what relationships existed between the three factors of labor, capital, and product.

If the relative supply from year to year of labor and capital were thus even approximately ascertained, a number of further problems would inevitably present themselves for solution of which the following are typical. (1) Can we estimate, within limits, whether this increase in production was purely fortuitous, whether it was primarily caused by technique, and the degree, if any, to which it responded to changes in the quantity of labor and capital?

(2) May it be possible to determine, again within limits, the *relative* influence upon production of labor as compared with capital?

(3) As the proportions of labor to capital changed from year to year, may it be possible to deduce the relative amount added to the total physical product by each unit of labor and capital and what is more important still by the *final* units of labor and capital in these respective years?

Might at least an historical approach to the theories of decreasing imputed productivity (diminishing increment to total product) be afforded and the way opened for further attempts to secure quantitative approximations to these assumed tendencies, if indeed there should turn out to be historical validity to them?

(4) Can we measure the probable slopes of the curves of incremental product which are imputed to labor and to capital and thus give greater definiteness to what is at present purely an hypothesis with no quantitative values attached?

(5) Finally from such a study of (a) the imputed physical product from year to year of a unit of labor and capital when joined with (b) a study of the relative exchange value of a physical unit of manufactured goods in these years and compared with (c) the actual movement of "real" wages in manufacturing and of real interest (if the latter can be ascertained), may we secure light upon the question as

[1] Mr. Douglas has been responsible for sections 1-5 and 8-10, of this paper and Mr. Cobb for sections 6 and 7.

to whether or not the processes of distribution are modeled at all closely upon those of the production of values?

The paper which follows attempts to deal with these questions and to throw some light upon them. But before this is done, it is of course necessary to construct indexes of the relative amounts of labor and capital which have been used and it is this which is dealt with in the two succeeding sections, leaving the later sections for the treatment of the interrelationships which may be discovered.

2. *The Growth of Fixed Capital in Manufacturing in the United States, 1899-1922.*—The census of manufactures has periodically included a question on the amount of capital invested in the various manufacturing enterprises and has tabulated the returns. This, however, includes in addition to fixed capital in the form of machinery and buildings, working capital including raw materials, goods in process of manufacture and finished goods in warehouses. It also includes land. Since we are attempting to measure the capital which aids in the production of goods, we should exclude working capital, for this is the result and not a cause of the process of manufacture.[2] We should also exclude land values since these are largely composed of the unearned increment. We shall therefore attempt to measure the changes in the physical quantity of (1) machinery, tools, and equipment and (2) factory buildings.

Unfortunately while statistics of total capital are given virtually every census year, they were only segregated for these specific groups in 1889, 1899, and 1904.[3] The Census Bureau in its 1922 report on *Wealth, Public Debt and Taxation* estimated that manufacturing machinery, tools, and equipment formed 30 per cent of the total amount of manufacturing capital.[4] Since it set the latter at 52,610 millions, this would give a figure for machinery, etc., of 15,783 million.

Year	Value of Factory Buildings (in millions of dollars)	Percentage of Manufacturing Capital	Value of Machinery, Implements and Equipment (in millions of dollars)	Percentage of Total Manufacturing Capital
1889.....	879	13.4	1,584	24.3
1899.....	1,450	14.8	2,543	25.9
1904.....	1,996	15.8	3,490	27.5
1922.....			15,783	30.0[5]

[2] Working capital of course normally "produces" value for its owner but we are here not concerned with value but with physical production.

[3] See *13th Census* (1900), VI, xcvii, and the *Census of Manufactures*, 1904, Part I, pp. lxiv-lxv.

[4] Bureau of the Census, Estimated National Wealth (1925), pp. 9-10.

[5] Estimate of the Census Bureau.

The amounts which have thus been ascribed to each of these forms of capital and the percentages which they formed of total capital for the given years were as shown on page 140.

These statistics furnish a basis for estimating the probable value of these forms of manufacturing capital in those years when no such segregation of items was carried out. Not only was the total amount of capital increasing but fixed capital was coming to form a larger percentage of this greater sum.

It seems undeniable that buildings and machinery did not increase as rapidly in comparison with working capital during the eighties as they did during the fifteen years which followed 1889 when buildings advanced from 13.4 to 15.8 per cent, or an increase of 2.4 points, and machinery, etc., from 24.3 to 27.5, or a gain of 3.2 points. This was an advance of .16 and .21 points a year, respectively. We have assumed the growth in the proportions which buildings formed of the total was at approximately only one-quarter of the rate of speed of the nineties and for machinery at only one-fifth. This would give 13.0 per cent as the probable figure for buildings in 1879 and 24.0 per cent as that for machinery, tools, and equipment.

If we accept the census estimate of 30 per cent as the proportion which machinery formed of the total in the terminal year of 1922, we may then distribute the 2.5 per cent increase from 27.5 per cent in 1904, according to a fairly even ratio. The rate of growth from 1914 on was, however, undoubtedly somewhat more rapid than during the previous decade and allowance should be made for this fact.

The growth in the relative importance of buildings since 1904 is more problematical since we have no end value on which to build. While the absolute increases have been enormous, it has not seemed to us that the relative importance of buildings in comparison with other forms of capital has advanced at the same rate as during the years 1889-1904. Because of this and the results of a Missouri investigation, we have estimated the percentage at 16.5 for 1922 and have distributed this over the preceding years but providing for a more rapid growth after 1914 than before. Table I gives the estimated percentage of each of these forms of manufacturing capital in the various years and the amounts in terms of dollars.

There is some evidence to indicate that the estimated total for buildings and machinery at 46.5 per cent is not far from correct. Thus the Missouri State Bureau of Labor Statistics shows that in 1923, 334.7 millions were invested in that state in manufacturing buildings, machinery, etc., and 58.7 millions in "grounds."[6] The amount of the working capital is not given but this was set for the country as a

[6] *Forty-fourth Annual Report Missouri Bureau of Labor* (1923), p. 155.

whole by the Federal Trade Commission at 45.7 per cent of the total capital.[7] Since this is based upon the returns of 54,862 corporations with a total capital of 33.65 billions, it may be accepted as the best nation-wide estimate which we have. If we apply this ratio to Missouri, we would get 331.1 millions or a total with other items of 724.6 millions. Now buildings, machinery, and equipment were, as stated, evaluated independently by the Missouri study at 334.7 millions and this would be 46.2 per cent of the total. This is in almost exact agreement with the estimate of 46.5 per cent which we have made for these forms of capital in 1922. Since the types of manufacturing in Missouri are not unrepresentative[8] of conditions in the country as a whole, our estimate can be considered to be buttressed and until better statistics are collected to be probably the best which can be made.

TABLE I

ESTIMATED VALUES OF MANUFACTURING BUILDINGS AND MACHINERY, TOOLS AND EQUIPMENT AND PERCENTAGES WHICH THEY FORMED OF TOTAL MANUFACTURING CAPITAL, 1879–1922

Year	Percentage of Total Manufacturing Capital		Value in Millions of Dollars		
	Buildings	Machinery and Equipment	Buildings	Machinery and Equipment	Total
1879.....	13.0	24.0	363	670	1,033
1889.....	13.4	24.3	879	1,584	2,463
1899.....	14.8	25.9	1,450	2,543	3,993
1904.....	15.8	27.5	1,996	3,490	5,486
1909.....	16.0	28.1	2,948	5,178	8,126
1914.....	16.2	28.7	3,692	6,541	10,233
1919.....	16.4	29.5	7,293	13,118	20,411
1922.....	16.5	30.0	8,681	15,783	24,464

There remains however the natural query as to what these census returns mean and how much the original data are worth. In recent years, the Census Bureau has instructed its agents to see that these statistics be taken "at the amounts carried on the books." Does this book value then mean the original cost of the buildings, machinery, etc., or the cost of reproduction? Mr. La Verne Beals, the chief statistician for manufactures, who is probably the ablest expert in this general field, has stated[9] that the "manufacturers have as a rule reported capital on the basis of original cost rather than cost of reproduction."

[7] Federal Trade Commission, *National Wealth and Income,* p. 135. (Senate Doc. 126, 69th Congress, 1st Session.)

[8] Thus while Missouri does not have any textile industries and but a small clothing industry, it does have a considerable amount of capital invested in printing, foundries, automobile manufacture, meat packing, smelting, and brick and lime works. There is also a fast growing shoe industry.

[9] Letter to author, October 23, 1925.

It is true that the Census Bureau has frequently issued cautions against accepting too implicitly its total for manufacturing capital and has indeed omitted such a question from its schedule for the 1921, 1923, and 1925 censuses. But, if the difficulties created by the fact that the investments are computed in terms of the price levels of the different years in which they were originally made can be overcome, and if the capital index can then be reduced to dollars of constant purchasing power, there would then seem to be no good reason why the resulting data should not be taken as a fairly accurate index of the *relative growth* of fixed capital, if not of its absolute amount. Moreover the proper correction of the distortions produced by changing price levels would remove most of the objections which can be leveled against such figures as a measurement of the total amount of capital. There remain two further problems before we can construct a continuous and comparable index: (1) finding the probable increments in each of the intervening years and (2) reducing these various increments of savings in terms of the value of a common price level.

Since the statistics are based upon original cost, the first problem consists in finding the annual increments of capital in terms of the prices of that year and of adding these to the values of the preceding year. The method followed was, in brief: (1) To ascertain the quantities of the following commodities produced in each year from 1899-1922: pig iron, rolled and forged steel, lumber, coke, cement, bricks and copper.[10] It will be noted that these commodities are the most important of those which are used in the construction of machinery and of buildings. In those few cases where it was impossible to secure actual figures of production for a given year, these were estimated from other years on the basis of the relative change in Professor Day's index of physical production for that group of manufactured products in which the commodity in question was included.[11] For the period 1880-1889, the quantities of pig iron, steel, cement, copper and coke were used. (2) The quantity produced of each commodity in each year was multiplied by its current price per unit of product.[12] The prices for the period from 1890-1922 were those collected and published by the United States Bureau of Labor Statistics[13] while those used for the decade from 1880-1890 were those published in the reports of the

[10] The raw data were secured from the United States Statistical Abstract for the various years. Also *Mineral Resources of the United States* 1921 Part I, pp. 235-82; 565-98; Part II, pp. 371-440.

[11] E. E. Day, "An Index of the Physical Volume of Production," *Review of Economic Statistics,* Vol. II (1920) pp. 328-29. Day, "The Physical Volume of Production in the United States for 1923." Ibid. Vol. VI (1924) p. 201.

[12] The average of the prices of spruce and maple was used for lumber.

[13] Bulletin 335 of the United States Bureau of Labor Statistics, *Wholesale Prices,* 1890-1922, pp. 126-56.

Aldrich Committee.[14] In some cases, it was possible directly to derive the value of the total product without multiplying the physical product by the price per unit and wherever this was the case the directly quoted total was used. (3) The values of each commodity produced in a given year were then added together to obtain the total values of these producers goods turned out in each year. (4) The values of these capital goods which were produced between two census years were then totaled (e.g., 1880 to 1889 inclusive) and the value for each year was divided by the total for the period in order to get the percentage which it formed of the total value produced during the period as a whole. These percentages were then applied to the total increase in the value of buildings and machinery over the same period and estimated yearly increases in the value of these items were thus obtained.

This process may be illustrated by the following example. The increase in the value of buildings and machinery between 1879 and 1889 was 1430 millions. The total money values in each of the years of these capital goods and the per cent which each of these yearly totals formed of the total for the period were as follows:

Year	Value of Specified Capital Goods (in millions of dollars)	Percent of Total Value for Decade
1880	200	9.6
1881	210	10.0
1882	216	10.3
1883	184	8.8
1884	148	7.1
1885	141	6.7
1886	211	10.0
1887	282	13.5
1888	241	11.5
1889	263	12.5
Total	2,096	100.0

The increase in the value of buildings and machinery during the decade, 1430 millions, was then multiplied by each of these percentages and the probable amounts of the yearly increases in value were obtained. These amounts when totaled and added to the total for 1879 would of necessity equal the 1889 value. The basic assumption is of course that the capital values in terms of original cost grew from year to year as the money value of the capital goods produced.

But since these estimated additions to capital are reckoned in terms of the dollars of the given years, if we are to secure an index of rela-

[14] *Report of Senate Committee on Whole Prices, on Wages and on Transportation*, Appendix A. The criticisms of the index of prices do not apply here since the absolute prices quoted were used.

tive real capital, it is necessary to eliminate the effect of changing price levels. A capital cost index was accordingly computed which was based on three sets of relative prices: (1) the wholesale prices of metals and metal products, (2) the wholesale prices of building materials and (3) money wages. The Aldrich Committee report was used to obtain prices for the first two groups of products from 1880 to 1889[15] while the indexes of the Bureau of Labor Statistics were used for the years 1890 to 1922.[16] For wages, the index previously computed by one of the authors was used for the period from 1890 on[17] while the average wages computed by Dr. R. P. Falkner for the Aldrich report were taken to show the movement during the eighties. These three series were then reduced to relatives with 1880 serving as 100 and were

TABLE II

ESTIMATED ANNUAL ADDITIONS TO FIXED CAPITAL IN MANUFACTURING TOGETHER WITH CUMULATIVE TOTAL CAPITAL AS EXPRESSED IN TERMS OF COST AND 1880 PRICES
(Millions of dollars), 1899–1922

Year	Annual Increase in Terms of Cost Price (1)	Cost Index (1880=100) (2)	Annual Increase in Terms of 1880 dollars (3)	Total Fixed Capital in 1880 dollars (4)	Relative Total Capital 1899=100 (5)
1899.....	339	88	387	4449	100
1900.....	264	89	297	4746	107
1901.....	277	88	315	5061	114
1902.....	342	89	383	5444	122
1903.....	328	91	362	5806	131
1904.....	282	87	326	6132	138
1905.....	457	92	494	6626	149
1906.....	612	100	611	7237	163
1907.....	629	106	595	7832	176
1908.....	373	94	397	8229	185
1909.....	569	96	591	8820	198
1910.....	422	100	420	9240	208
1911.....	379	99	384	9624	216
1912.....	457	103	443	10067	226
1913.....	497	110	453	10520	236
1914.....	356	101	353	10873	244
1915.....	1017	105	967	11840	266
1916.....	1899	135	1402	13242	298
1917.....	2891	173	1673	14915	335
1918.....	2473	183	1350	16265	366
1919.....	1898	196	969	17234	387
1920.....	2096	237	884	18118	407
1921.....	780	184	424	18542	417
1922.....	1177	181	650	19192	431

[15] *Report of Senate Committee on Wholesale Prices*, etc., pp. 92-99. The celebrated twenty-five varieties of jack-knives were subtracted from the metal index before using it.

[16] Bulletin 335, *Wholesale Prices*, 1890-1922, pp. 8-9.

[17] Paul H. Douglas, "The Recent Movement of Real Wages and Its Economic Significance." Supplement, *American Economic Review*, March, 1926, p. 30.

combined into a weighted average. The weights used were metals and metal products, 4; building materials, 2; and wages, 3.

Each yearly increase in the value of manufacturing buildings and machinery was then divided by the relative cost index for that year and a series of "deflated" increases were thus obtained, or rather a series of increases which were expressed in terms of the 1880 price level for capital goods. The next and final step was to add these deflated yearly increases to the estimated total for buildings and machinery for 1879 and thereafter to the total for each preceding year. Table 2 shows all this material. Since our other data only extend from 1899-1922, the years prior to 1899 are omitted from this table. Values given are in millions of dollars.

The index is defective in that it does not allow for the replacement of original capital at differing price levels. The census statistics of book value, undoubtedly include replacements made at different and generally higher prices than those which prevailed when the original capital was invested. Consequently the advance from year to year is not solely the result of the saving of additional increments of capital but includes in part the replacement at other price levels of the old capital as it wore out. The consequence is that our index is throughout most of its course somewhat higher than it should be. It is hoped to publish a revision of this index in the not distant future in which this error will be eliminated. In the meantime this is offered as a first approximation.

The index does not of course measure the short-time fluctuations in the amount of capital used. Thus, no allowance is made for the capital which is allowed to be idle during periods of business depression nor for the greater than normal intensity of use in the form of second shifts, etc., which characterizes the periods of prosperity.

The validity of this index of growth is somewhat strengthened, however, when we compare the increase in terms of book value which we have estimated for the United States[18] during the years 1910-1920 with

Year	Massachusetts (Total Capital)	Estimated for United States (Fixed Capital)
1911	105	104
1912	110	110
1913	113	116
1914	130	120
1915	130	132
1916	150	154
1917	188	188
1918	210	217
1919	248	239
1920	250	263

[18] This column was omitted from Table II because of lack of space.

the growth of total capital in Massachusetts when computed upon a similar basis.[19] Using 1910 as a base, the relative increases were as shown on page 146.

The coincidence between these two indexes is very striking and this becomes even more the case when we remember that most of the greater increase shown for the United States as a whole was due to the fact that the fixed capital was increasing at a more rapid rate than was the supply of total capital in manufacturing.

It may be remarked that this index shows a truly unprecedented growth in the volume of fixed capital. Thus the amount virtually doubled during the decade from 1899-1909. This was a compounded average yearly rate of increase of 7 per cent. This same rate of increase was virtually maintained during the succeeding decade. From 1919 on the rate of growth slackened during the three succeeding years but while we have not computed the growth since 1922 it has beyond question increased greatly since then. Taken as a whole this period showed an approximate doubling in the quantity during every decade, which would probably be scaled down to about 6 per cent per year compounded if deductions were made for the increased cost of replacing the old capital. This is a rate of growth which it is believed has not been matched by any other country.[20] It will be remembered that Cassel estimates the rate of growth of capital in Western Europe at 3 per cent a year. If this is true, the rate of industrial capital growth in the United States has been twice as great while if the growth be reckoned on a per capita basis, the disparity is even greater.

3. *The Growth in the Labor Supply, 1899-1922.*—The various censuses of manufactures give the average number of wage-earners employed in each of the census years.[21] Using these as the bases, we can find the probable numbers employed in the intercensus years by using an index of relative employment. This index was constructed for the years 1899-1904 by combining statistics of the relative number employed from year to year in Massachusetts[22] and Pennsylvania.[23] From 1904 to 1914, figures for New Jersey[24] were substituted for those of Pennsylvania. In both periods, the relative index for each state was then weighted by the number shown by the census to be employed in that state at the beginning of the period and a combined index was

[19] See Annual reports of Massachusetts Bureau of Statistics, *Statistics of Manufactures,* 1910-20.

[20] Our index shows a more than doubling from 1879 to 1899 and an increase of approximately 90 per cent during the nineties.

[21] I.e. namely 1899, 1904, 1909, 1914, 1919, and 1921.

[22] See Annual reports on *Statistics of Manufactures, Massachusetts,* 1900-1905.

[23] See Reports Pennsylvania State Department of Internal Affairs.

[24] Annual volume of New Jersey Bureau of Labor and Industries, *Statistics of Manufactures* (1904-1914).

thus secured. The assumption was then made that the volume of employment of the country as a whole followed a similar course to that in these two states. When the rate of change in these two states differed over a census period from the country-wide figures, then it was assumed that this greater or less degree of change had been distributed evenly over the intervening years, and the percentage changes for the two states were scaled down or up to conform to this standard.[25] Thus the increase in the number employed in 1904 over 1899 was as shown by the Census 1,066,000, or 21 per cent. If the increase shown for Massachusetts and New Jersey was 24 per cent, then it was assumed that the differences between the rate of growth for the country and for the two states increased annually at the rate of one-fifth of 3 per cent or .6 per cent. Then if the increase shown in Massachusetts and New Jersey for 1900 over 1899 was 4.6 per cent, this was scaled down to 4.0 per cent. Similar methods were used for the subsequent years.

From 1914 to 1919 the index was secured by combining that of the Bureau of Labor Statistics[26] for a number of industries with that for New York. In doing this, the Bureau's index was given a weight of 3 and that of New York a weight of 1.[27] From 1919 on, the index of the Federal Reserve Board was used which in turn was largely based upon the index of the Bureau of Labor Statistics. A substantially similar method was used to find the probable number employed in each

TABLE III

THE PROBABLE AVERAGE NUMBER OF WAGE-EARNERS EMPLOYED IN MANUFACTURING 1899–1922

Year	Average Number Employed (in thousands)	Relative Number (1899=100)	Year	Average Number Employed (in thousands)	Relative Number (1899=100)
1899.....	4713	100	1911	6855	145
1900.....	4968	105	1912	7167	152
1901.....	5184	110	1913	7277	154
1902.....	5554	118	1914	7026	149
1903.....	5784	123	1915	7269	154
1904.....	5468	116	1916	8601	182
1905.....	5906	125	1917	9218	196
1906.....	6251	133	1918	9446	200
1907.....	6483	138	1919	9096	193
1908.....	5714	121	1920	9110	193
1909.....	6615	140	1921	6947	147
1910.....	6807	144	1922	7602	161

[25] This is the identical method which I have followed in interpolating average annual earnings in the intercensal years from the statistics of earnings of the various states.

[26] See files of *Monthly Labor Review.*

[27] See New York *Labor Market Bulletin.*

of the intercensal years up to and including 1922.[28] Table III gives these estimated numbers from 1899 on and also expresses them in terms of relatives.

This index is defective in a number of respects as a perfect measure of the working force. (1) It does not include clerical employees who have been increasing in number at approximately double the rate of the wage-earners. (2) It is based on man-years rather than "standard" man hours. The average number of hours constituting the standard week's work has declined during this period, so that an increase in the number of men would be necessary merely to offset this decrease. One of the authors has computed a tentative index of standard man hours by multiplying the number of workers in each year by the average number of hours in the "normal" week. There is reason to believe, however, that this index is not yet perfected and so man-years have been used instead. It is hoped to include total "standard" hours in later studies. (3) It does not measure deviations from this standard week whether they take the form of short-time in periods of depression or overtime in the years of prosperity.

Such an index of course makes no allowance for possible changes in the quality of the laborers or in the intensity of their work. These factors may be of considerable importance but at present they certainly cannot be measured quantitatively and until they can be, it is better for any statistical study to ignore them than to make necessarily fantastic estimates as to their importance. When they can be measured, then they should be included.

4. *The Growth of Physical Production, 1899-1922.*—For this, we have used E. E. Day's well-known index of the physical volume of

TABLE IV

INDEX OF PHYSICAL VOLUME OF MANUFACTURES IN THE UNITED STATES

Year	Index of Manufactures	Year	Index of Manufactures
1899	100	1911	153
1900	101	1912	177
1901	112	1913	184
1902	122	1914	169
1903	124	1915	189
1904	122	1916	225
1905	143	1917	227
1906	152	1918	223
1907	151	1919	218
1908	126	1920	231
1909	155	1921	179
1910	159	1922	240

[28] Since these statistics of employment did not begin until July, 1914, the yearly average was secured by projecting them back for the preceding six months according to the monthly fluctuations in employment shown by the 1914 census of manufactures.

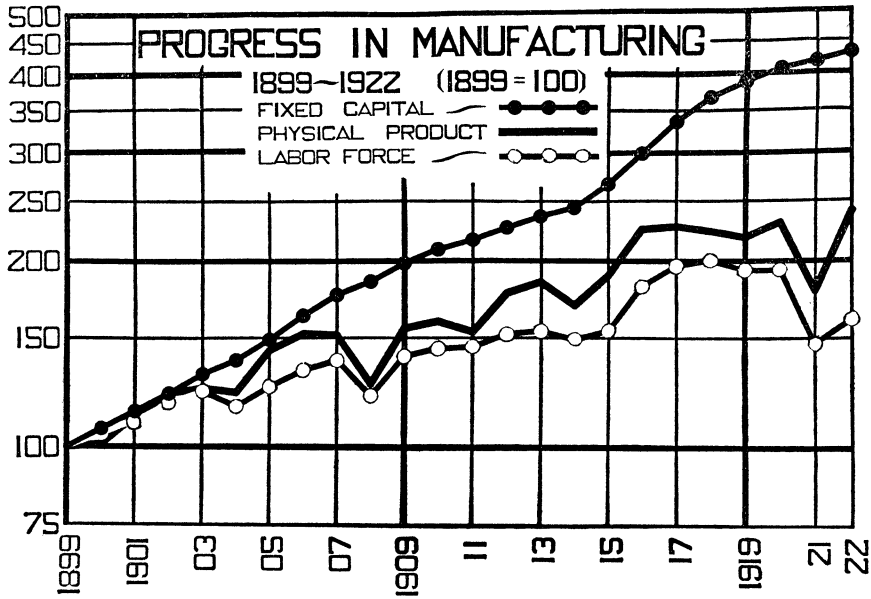


CHART I

production for the years 1899-1922, since at the time we were carrying through our studies the later index given by Dr. Thomas was not available.[29]

Chart I shows on a logarithmic scale the relative growth in manufacturing during this period of fixed capital, of the labor force, and of the

TABLE V

THE RELATIVE PROPORTIONS OF LABOR AND CAPITAL WHICH WERE COMBINED IN MANUFACTURING 1899–1922 (1899=100)

Year	Relative Amount of Labor to Capital	Year	Relative Amount of Labor to Capital
1899	100	1911	67
1900	98	1912	67
1901	96	1913	65
1902	97	1914	61
1903	94	1915	58
1904	84	1916	61
1905	84	1917	59
1906	82	1918	55
1907	78	1919	50
1908	65	1920	47
1909	71	1921	35
1910	69	1922	37

[29] For a description of the methods and sources used in computing the index of production for manufactures, see E. E. Day and W. M. Persons, "An Index of the Physical Volume of Production." *Review of Economic Statistics,* II (1920) 309-37; 361-67. See also Ada M. Mathews, "The Physical Volume of Production in the United States in 1924," *Ibid.,* VII. (1925), 215.

physical product. It will be noted that by 1922 the supply of capital had more than quadrupled as compared with 1899, while the labor force was only 61 per cent greater. The ratio of capital to the working force was indeed 2.67 times as great in 1922 as it had been in 1899. The increase in the physical product during this period was 140 per cent or an increase of approximately 50 per cent per worker.[30]

5. *The Ratio of Labor to Capital.*—The changing ratio between labor and capital as compared with 1899 can be found by dividing the relative index of the labor supply by the relative index of fixed capital (L/C). This is shown in Table V. We thus have a measure of the changing proportions of the two factors throughout the years of this period.

It will be noted that since our index of labor measures the decline in the number of wage-earners employed during periods of depression while our index of capital does not show the unused capital, that during such years, the proportion of labor to capital drops sharply, with a tendency to rise during the succeeding years. The general drift is, however, of course downward because of the much more rapid increase of capital.

6. *Theory of Production.*—Relative to the indices of Production, Labor, and Capital, and the period 1899–1922 the function of Labor and Capital alone

$$P' = 1.01 L^{3/4} C^{1/4}$$

has the following properties:

1) To say that P' represents the actual Production P is to give particular expression to a well-known theory.
2) P' approaches zero as either L or C approaches zero.
3) P' approximates P over the period.
4) The deviations of P' from P are individually significant.
5) P' correlates closely with P when we include secular trends.
6) P' correlates closely with P when we eliminate secular trends.

In the sense of the foregoing let us call P' a *norm* for P over the period, and proceed to examine its properties in more detail.

(1) The theory referred to (due to J. B. Clark, Wicksteed et al.) states that Production, Labor and Capital are so related that if we multiply both Labor and Capital by a factor m then Production will be increased m times, that is Production is a first degree homogeneous function of Labor and Capital. Now P' is taken to be such a function.

(2) Among such functions the further theoretical restriction is placed upon P' that it should approach zero as either L or C approaches zero.

[30] As Dr. Thomas shows, the most remarkable increase in productivity has come since 1921 and is scarcely included in the above statistics.

Among functions with these properties (1) and (2) let us make a definite choice[31] and examine the consequences of that choice, reserving the right to make other choices if we wish. Let us choose the function

$$P' = bL^kC^{1-k}$$

and find such numerical values of b and k that P' will "best" approximate P in the sense of the Theory of Least Squares. Then relative to the indices and the period we have the norm

$$P' = 1.01L^{3/4}C^{1/4}$$

(3) Given the indices of L and C, the function P' may be computed and may be compared with P in Table VI and Chart II as follows:

TABLE VI

RELATION BETWEEN (1) PRODUCT CALCULATED FROM RECORDED VALUES OF L AND C (TABLES II AND III) BY MEANS OF THE FORMULA $P' = 1.01\ L^{3/4}\ C^{1/4}$ AND (2) RECORDED VALUES OF PRODUCT (TABLE IV)

Year	P' Product Calculated (1)	P Product Recorded (2)	Percent Deviation $\frac{(2)-(1)}{(2)}$	Business Annals[32]
1899	101	100	−1	Prosperity
1900	107	101	−6	Prosperity; brief recession
1901	112	112	0	Prosperity
1902	121	122	+0.8	Prosperity
1903	126	124	−1.6	Prosperity; recession
1904	123	122	−0.8	Mild depression; revival
1905	133	143	+7.	Prosperity
1906	141	152	+7.	Prosperity
1907	148	151	+2.	Prosperity, panic, recession, depression
1908	137	126	−9.	Depression
1909	155	155	0	Revival, mild prosperity
1910	160	159	−0.6	Recession
1911	163	153	−6.5	Mild depression
1912	170	177	+4.	Revival; prosperity
1913	174	184	+5.5	Prosperity; recession
1914	171	169	−1.2	Depression
1915	179	189	+5.	Revival; prosperity
1916	209	225	+7.2	Prosperity
1917	227	227	0.	Prosperity; war activity
1918	236	223	−6.	War activity; recession
1919	233	218	−7.	Revival; prosperity
1920	236	231	−2.2	Prosperity; recession, depression
1921	194	179	−8.4	Depression
1922	209	240	+13.	Revival; prosperity

[31] This amounts to an assumption that the marginal productivity of labor is proportional to the production per unit labor and the marginal productivity of capital is proportional to production per unit capital. These properties are derived from the "chosen" function in a later section.

[32] W. L. Thorp *Business Annals*, p. 138 ff.

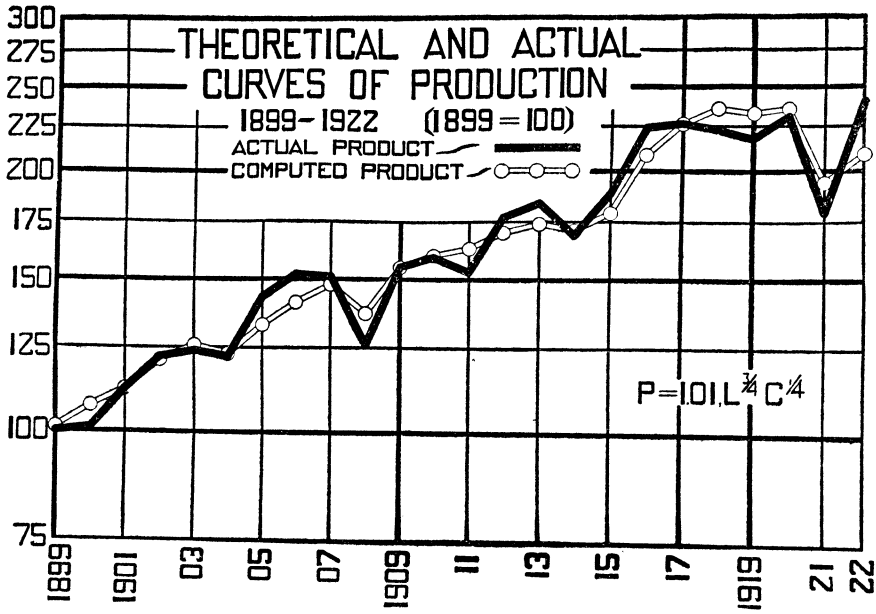


CHART II

The average percentage deviation of P' from P without regard to sign is 4.2 per cent. In fact, P lies nearer to P' than to its own moving three year average, the corresponding standard deviations being 8.7 and 11.7 respectively.

TABLE VII

DEVIATIONS FROM TREND OF P AND P′ (TRENDS ARE MOVING 3 YEAR AVERAGES)

Year	Deviation of P from Trend of P	Deviation of P′ from Trend of P′	Year	Deviation of P from Trend of P	Deviation of P′ from Trend of P′
1900.....	−3	0	1911.....	−10	−1
1901.....	0	−1	1912.....	6	1
1902.....	3	1	1913.....	7	2
1903.....	1	3	1914.....	−12	−4
1904.....	−8	−4	1915.....	−5	−7
1905.....	4	1	1916.....	11	4
1906.....	3	0	1917.....	2	3
1907.....	8	6	1918.....	0	4
1908.....	−18	−10	1919.....	−6	−2
1909.....	7	4	1920.....	22	15
1910.....	3	1	1921.....	−38	−19

(4) It is evident from the foregoing Table VI and Chart II that the trends of P and P' (say the moving three year averages) are much alike, in fact P' was constructed so that they should be. A study of Table VII and Chart III will show also that in general P' fluctuates with the business

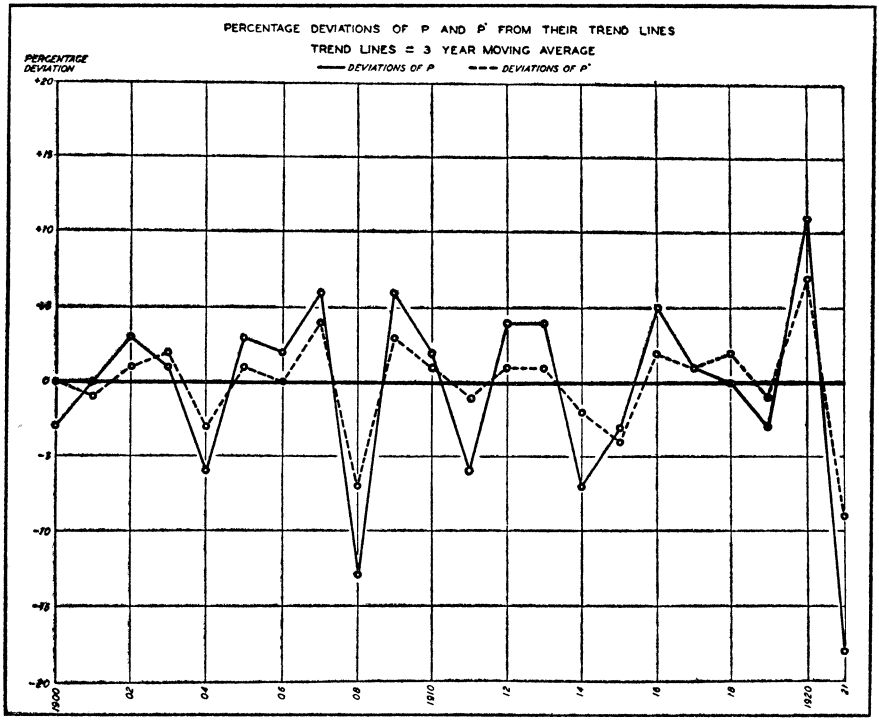


CHART III

cycle in the same direction as does P, with this difference that the oscillations of P' (relative to trend) are not as great as those of P, due to the steadying influence of the steadily increasing C.

When we consider also the Business Annals as given by W. L. Thorp it is evident (Table VI and Chart IV)[33] that in general we compute too little in times of prosperity and too much in times of depression. Then not only does P' follow the business cycle but also the deviations of P' from P follcw the business cycle.

(5) and (6). The coefficient of correlation between P and P' with trends included is .97 and with trends eliminated is .94.

So far we have been taking for granted that the "normal" production P' would have been produced with given quantities of labor and capital under "normal" conditions. These normal conditions are fictitious. For example the productive power of the "average" worker or of the dollar of constant purchasing power is supposed to remain constant over the period. For normal conditions management would not be more or less efficient at different times. There would be no booms nor depressions, no wars and

[33] Note that the algebraic signs of the percentages in the chart are opposite from those in the table.

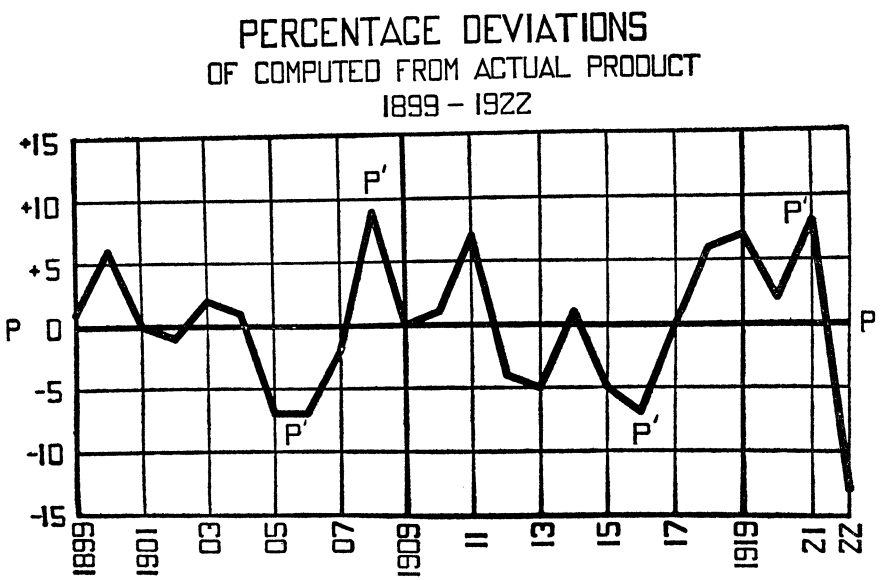


CHART IV

so on, under normal conditions. The differences between production under normal conditions and production under actual conditions may be compared as in (4) above with the *Business Annals* of the period, year by year.

Now it is possible to apply mathematical analysis to the fictitious production P' but it is not possible to apply such analysis to the actual production P unless we make (or conceal) certain further assumptions. Let us choose the following assumptions and let their justification rest on what we deduce from them.

(A) The Physical Volume of Production is proportional to the Volume of Production due to manufacturing alone.

(B) Any departure of P from P' may be represented by a change in the value of the coefficient of $L^{3/4}C^{1/4}$ so that always

$$P = bL^{3/4}C^{1/4}$$

where the value of b is independent of L and C.

These two assumptions are made in accordance with a general policy to ignore the quantitative effects of any force for which we have no quantitative data. The coefficient b is thus made a catch-all for the effects of such forces.

Making these assumptions it follows at once by mathematical analysis that:

I. The marginal productivity of labor is $3/4\ P/L$.

II. The marginal productivity of capital is $1/4\ P/C$.

III. The productivity of total labor is $3/4P$.

IV. The productivity of total capital is $1/4P$.

This imputes three-fourths of the product to labor and one-fourth to capital for the period in question.

V. The elasticity of the product with respect to small changes in labor alone is 3/4.

VI. The elasticity of the product with respect to small changes in capital alone is 1/4.

This means that a small percentage change in labor alone has three times the effect that would be made by the same small percentage change in capital alone.

These six theorems will be proved in the next section. It should be born in mind, however, that our results have been given exact numerical values for the sake of fixing the ideas. But the numbers themselves are fixed tentatively relative to a certain period and to certain indices. When the indices are refined or the period is changed it may be that the constant 3/4 will appear as a constant .7 or .6 or perhaps as a variable. Even the *form* of the function P' may have to be changed.

It is the purpose of this paper, then, not to state results but to illustrate a method of attack. In choosing a definite Norm for Production as a first approximation it is not at all certain that we have arrived immediately at the best possible. The advantage in choosing a norm at all seems to be that it involves us in logical consequences which may be compared with the facts as we get the facts. It enables us to talk rightly or wrongly with more precision and to draw conclusions which become hypotheses.

7. *Mathematical Analysis.*—Given the function

$$P = bL^k C^{1-k}$$

where b is independent of L and C and (to fix the ideas) k is supposed to be constant and equal to 3/4. Then the six theorems of the preceding section may be proved by the six equations which follow:

(1) $$\frac{\partial P}{\partial L} = \frac{3}{4}\frac{P}{L}$$

(2) $$\frac{\partial P}{\partial C} = \frac{1}{4}\frac{P}{C}$$

(3) $$\frac{L\partial P}{\partial L} = \frac{3}{4}P$$

(4) $$C\frac{\partial P}{\partial C} = \frac{1}{4}P$$

(5) $$\frac{\partial(\log P)}{\partial(\log L)} = \frac{3}{4}$$

(6) $$\frac{\partial(\log P)}{\partial(\log C)} = \frac{1}{4}$$

If b is taken equal to 1.01 say, then

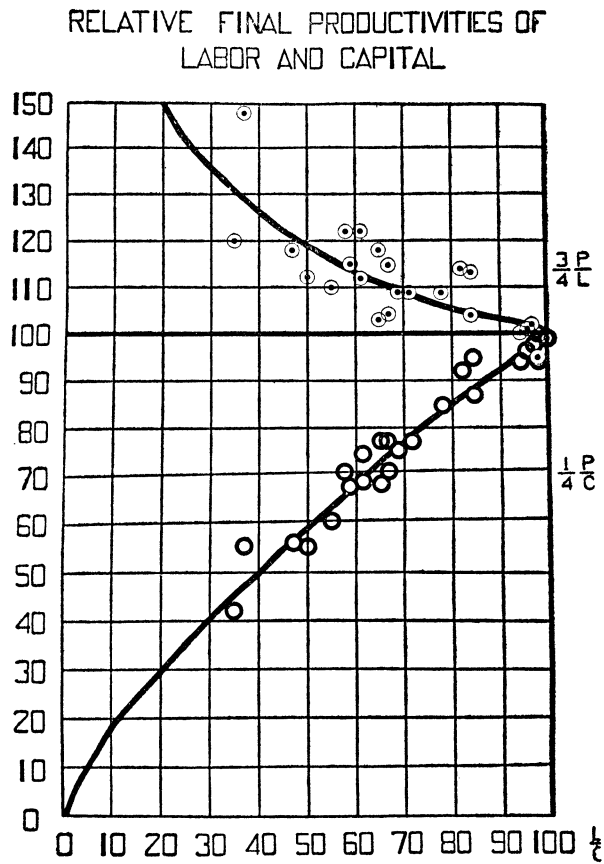
(7) $$\frac{\partial P}{\partial L} = 1.01 \times \frac{3}{4} \times \left(\frac{L}{C}\right)^{-1/4}; \quad b = 1.01$$

$$\frac{\partial P}{\partial C}=1.01\times\frac{1}{4}\times\left(\frac{L}{C}\right)^{3/4}\quad;\quad b=1.01 \tag{8}$$

From (7) and (8) it follows that just as Production has a norm which it approximates, so the marginal productivities of labor and capital have norms which they approximate; namely, the curves $y=1.01(L/C)^{-1/4}$ and $y=1.01(L/C)^{3/4}$ respectively.

The three norms and the corresponding quantities are so related that if one quantity, say production, rises above its norm by 5 per cent then each of the other two quantities rises above its norm by 5 per cent. This is due to the algebraic identity

Chart V[34]



[34] In the chart the "normal" curves are taken without the coefficient 1.01 and the indices of marginal productivity are depressed proportionally.

$$\frac{P}{L}:\left(\frac{L}{C}\right)^{-1/4}=\frac{P}{C}:\left(\frac{L}{C}\right)^{3/4}=P:L^{3/4}C^{1/4}=b:1.$$

We may now find the rates of change of the marginal productivities and total productivities by taking derivatives of equations (1) to (4) replacing the constant 3/4 by the indefinite k and remembering that k is to be constant, positive and less than 1.

$$\frac{\partial}{\partial C}\left[\frac{\partial P}{\partial L}\right]=k(1-k)\frac{P}{LC} \tag{9}$$

$$\frac{\partial}{\partial L}\left[\frac{\partial P}{\partial C}\right]=k(1-k)\frac{P}{LC} \tag{10}$$

and hence:

The productivity of unit labor *increases* per unit increase in capital alone. The productivity of unit capital *increases* per unit increase in labor alone. These rates of increase (which are equal for fixed values of L and C) are given by the expression on the right hand side of equations (7) and (8).

$$\frac{\partial}{\partial L}\left[\frac{\partial P}{\partial L}\right]=k(k-1)\frac{P}{L^2} \tag{11}$$

and hence (diminishing returns):

The productivity of unit labor *decreases* per unit increase in labor alone (since $k-1$ is negative) at a rate given by the right hand side of equation (11).

Similarly:

$$\frac{\partial}{\partial C}\left[\frac{\partial P}{\partial C}\right]=k(k-1)\frac{P}{C^2} \tag{12}$$

and hence (diminishing returns):

The productivity of unit capital *decreases* per unit increase in capital alone at a rate given by the right hand side of equation (12).

$$\frac{\partial}{\partial L}\left[L\frac{\partial P}{\partial L}\right]=k^2\frac{P}{L} \tag{13}$$

and hence:

The productivity of total labor *increases* per unit increase in labor alone, at a rate given by the right hand side of equation (13).

$$\frac{\partial}{\partial C}\left[C\frac{\partial P}{\partial C}\right]=(1-k)^2\frac{P}{C} \tag{14}$$

and hence:

The productivity of total capital *increases* per unit increase in capital alone at a rate given by the right hand side of equation (14).

$$\frac{\partial}{\partial L}\left[C\frac{\partial P}{\partial C}\right]=k(1-k)\frac{P}{L} \tag{15}$$

and hence:

The productivity of total capital *increases* per unit increase in labor alone at a rate given by the right hand side of equation (15).

$$\frac{\partial}{\partial C}\left[L\frac{\partial P}{\partial L}\right]=k(1-k)\frac{P}{C} \tag{16}$$

and hence:

The productivity of total labor *increases* per unit increase in capital alone at a rate given by the right hand side of equation (16).

Finally, if k is supposed to vary then P' becomes a function of three variables, and we have a new batch of theorems for example: "If k increases while L and C remain fixed then P' increases if L/C is greater than 1, and P' decreases if L/C is less than 1."

Thus if we choose a smaller k than 3/4 (say 2/3 for the whole period) the P' curve thus computed will lie above the P' curve computed with $k=3/4$ whenever L/C is less than 1, that is over most of the period. The relation between P and the new $P'=1.01L^{2/3}C^{1/3}$ is given in the following table.

TABLE VIII

RELATION BETWEEN P AND $P'=1.01\,L^{2/3}\,C^{1/3}$

	P	P′	$\frac{P-P'}{P}\times 100$		P	P′	$\frac{P-P'}{P}\times 100$
1899.....	100	101	−1	1911.....	153	166	−8
1900.....	101	106	−5	1912.....	177	173	+2
1901.....	112	111	+1	1913.....	184	178	+3
1902.....	122	119	+3	1914.....	169	176	−4
1903.....	124	125	−1	1915.....	189	185	+2
1904.....	122	123	−1	1916.....	225	214	+5
1905.....	143	133	+7	1917.....	227	234	−3
1906.....	152	142	+7	1918.....	223	244	−9
1907.....	151	149	+1	1919.....	218	243	−11
1908.....	126	139	−10	1920.....	231	247	−7
1909.....	155	157	−1	1921.....	179	208	−16
1910.....	159	163	−3	1922.....	240	223	+7

8. *What Indications are there That the Theory Outlined is Valid?*—That the equation $P'=1.01L^{3/4}C^{1/4}$ describes in a fairly accurate manner the actual processes of production in manufacturing during this period as indicated by:

(1) The close consilience between P and P' as shown in Table VI and Chart II with a coefficient of correlation of $+.97$. When three year moving averages of P and P' are taken the agreement is even closer, the percentage deviation of P' and P amounting on the average (without regard to sign) to only 2.6 per cent per year instead of 4.3 per cent on the year to year observations. The cumulative error of the three year moving average of P' from the three year moving average of P is in turn only $-.1$ per cent.

(2) The close degree to which the theoretical curves of imputed productivity of unit labor, i.e., $y=(L/C)^{-1/4}$ and of unit capital $(y=(L/C)^{3/4})$ form the curves of best fit to the "recorded" values of unit productivity of labor and of capital.

(3) It has some times been charged that the relationship discovered between capital, labor, and manufacturing product is purely fortuitous and that equally good results would be secured by comparing the relative movement of hogs in Wisconsin, cattle in Wisconsin, with the physical product in manufacturing. But there is a logical and economic connection between labor, capital, and product which is not present in the attempted *reductio ad absurdum*. Moreover the fact that the deviations of P' and P from their respective three year moving averages move closely together as is shown by Chart III and that they have a correlation coefficient of $+.94$ indicates that the relationship is not merely one between factors whose secular trend happens to be upward.

(4) The fact is that the deviations of P' from P are in nearly every case precisely what one would expect. Thus during depressions, large amounts of capital are of necessity allowed to lie idle but our index of capital growth makes no allowance for this. Similarly because of the practise of short-time, the number of man hours worked decreases by a greater ratio than that of the number of men employed. Our computed index P' would therefore be expected to be greater than the actual index P. Note then that in the depression years of 1908, 1911, 1914, 1920, and 1921, P' was 9, 7, 1, 2, and 8 per cent respectively higher than P, and that during the years marked by some recession or a slight depression such as 1900, 1903, 1904, and 1910, P' was also higher than P by 6, 2, 1, and 1 per cent respectively.

Conversely, since our index of labor does not take into account overtime hours nor our index of capital the greater intensity of use which prosperity brings, it would be expected that P' would be less than P during this phase of the cycle. This is born out in practice. For the prosperous years of 1905 and 1906, P' was 7 per cent below P, and for 1907, the first three-quarters of which displayed great activity, it was 2 per cent lower. In the prosperous years of 1912 and 1913, P' was in turn 4 and 6 per cent below P and in 1915 and 1916, again 5 and 7 per cent less than P. In 1922, P' was no less than 13 per cent below P.

The only two years which constitute an exception to what we would thus expect are 1918 and 1919. These were years of business prosperity yet P'

instead of being lower than P was actually higher by 6 and 7 per cent. This may, however, have been caused by the dilution and reduced efficiency of labor which made each unit of labor actually less productive than normally.

9. *Does the Process of Distribution Approximate the Apparent Laws of Production?*—We have attempted to check this theory to see whether the processes of distribution have followed in any degree the laws of production which we believe we have traced. By the methods which have previously been described (Sections 6 and 7) the relative final physical productivities of labor for each of the years were found in terms of 1899 to be as follows:

1899.....100	1907.....110	1915.....123
1900..... 96	1908.....104	1916.....123
1901.....102	1909.....110	1917.....116
1902.....103	1910.....110	1918.....111
1903.....101	1911.....105	1919.....113
1904.....105	1912.....116	1920.....119
1905.....114	1913.....119	1921.....121
1906.....115	1914.....113	1922.....149

These relative physical productivities were then multiplied by the relative exchange value of a composite unit of manufactured goods and thus the relative *value product* per laborer in each of the years as distinguished from the relative physical product was secured. It is then possible to compare the movement of this value product of final labor with the relative movement of the real wages of the workers during this period in order to determine the degree of correspondence between them.

Before entering upon such a comparison, however, it is appropriate to describe how the exchange ratio of each unit of manufactured *goods* and of manufacturing as a whole was found. This was secured by multiplying the index of physical production by the ratio between the price level of manufactured goods and the relative general price level.

$$\text{Index of physical production} \times \frac{\text{Price Index of Manufactured Goods}}{\text{General Price Level}}$$

This ratio of the prices of manufactured commodities to the general price level was computed from the statistics of wholesale prices collected by the Bureau of Labor Statistics and is shown in the following table.

This index shows that when measured in terms of 1899, a unit of manufactured goods had less purchasing power in ten subsequent years, reaching its lowest point of 85 in 1910. Its exchange value was somewhat higher in the subsequent years and it rose somewhat in 1922 when it was still 10 per cent below 1899. This in turn reduced the total

value product from 240 to 217 which although also shared by 1920, was the highest point for the period.

TABLE IX

RELATIVE VALUE PRODUCT OF MANUFACTURED GOODS AND TOTAL VALUE PRODUCT OF MANUFACTURING 1899–1922 (1899 = 100)

Year	Price of all Mfg Commodities 1	All Commodity Index 2	Ratio Mfg Commodities to all Commodities 3*	Total Value Product (Physical product times column 3)
1899	100	100	100	100
1900	105	108	98	99
1901	101	106	96	107
1902	103	113	91	111
1903	104	114	91	113
1904	103	114	90	109
1905	106	115	92	132
1906	112	118	95	144
1907	119	125	95	144
1908	110	120	91	115
1909	112	129	87	134
1910	115	135	85	136
1911	111	124	90	137
1912	116	132	88	156
1913	117	134	88	162
1914	113	131	86	146
1915	119	135	88	167
1916	156	169	92	207
1917	210	237	89	201
1918	226	259	87	194
1919	242	276	89	191
1920	284	302	94	217
1921	186	196	95	170
1922	179	199	90	217

* Column 3 equals column 1 divided by column 2.

The relative physical productivities of the final units of labor successive years were then multiplied by the relative exchange ratio of a unit of physical product for the appropriate year and an index of relative *value* productivity for the final units of labor in the various years was thus obtained. This was as shown in Table X with the average for the years from 1899 to 1908 taken as 100.[35]

This was then compared with the index of real wages for manufacturing computed by one of the authors.[36] To avoid the assumption that correlation was perfect in the year 1899, the average for the years

[35] The price statistics were taken from Bulletin 390 of the Bureau of Labor Statistics. The groups of commodities included to form the price index of manufactured goods were: (1) food, (2) cloths and clothing, (3) chemicals and drugs, (4) metals and metal products, (5) building materials, (6) house furnishings, (7) among the miscellaneous commodities, leather, paper and pulp, soap, and tobacco.

[36] Paul H. Douglas, "The Recent Movement of Real Wages and Its Economic Significance." Supplement, *American Economic Review*, March, 1926, p. 33.

1899-1908 was instead taken as the base. The comparative table is shown on page 164.

TABLE X

RELATIVE VALUE PRODUCTIVITY PER UNIT LABOR 1899–1922

Year	Relative Value Productivity per Unit of Labor	Year	Relative Value Productivity per Unit of Labor
1899	101	1911	96
1900	95	1912	103
1901	99	1913	106
1902	95	1914	98
1903	93	1915	110
1904	96	1916	115
1905	106	1917	104
1906	111	1918	98
1907	105	1919	102
1908	96	1920	114
1909	97	1921	117
1910	95	1922	136

The coefficient of correlation between these series is + .69 with a probable error of ± .072 and if a comparison is made between the seven year moving averages of the two, the coefficient is + .89 with a probable error of ± .03. There is virtually no relationship, however, between the short time movements of the two, since the correlation of the deviations of each from its trend gives a coefficient of only .12.

The degree of correspondence discovered is however sufficient to give a considerable degree of corroboration to the law of production which has been worked out and to indicate that the processes of distribution follow in large measure the processes of production if sufficient time is allowed.

A further interesting comparison is afforded by the studies of the National Bureau of Economic Research into the proportion of the manufacturing product which went to labor during the decade 1909-1918. They found that wages and salaries formed on the average 74 per cent of the total value added by manufactures during these years.[37] We have found in our formula that when we attribute to labor 75 per cent of the product, we get a close consilience to the actual normal course of production.

There is apparently therefore a decided tendency for distribution to follow the laws of imputed productivity. Lest some be led however hastily to conclude that this lends an ethical justification to the existing social and economic order, it should be pointed out that

[37] National Bureau of Economic Research, *Income in the United States*, Vol. 2, p. 98. The percentages by years were as follows:

1909	72.2	1911	76.4	1913	74.5	1915	75.4	1917	71.0
1910	71.6	1912	74.5	1914	77.8	1916	68.7	1918	78.1

TABLE XI

RELATIVE MOVEMENT IN MANUFACTURING OF IMPUTED VALUE PRODUCT PER WORKER AND REAL WAGES (1899–1922) (1899–1908 = 100)

	(1) Value Product Unit Labor (Average 1899–1908 = 100)	(2) Real Wages (Average 1899–1908 = 100)	(3) Per Cent Deviation of (2) from (1). $\frac{(2)-(1)}{(1)}$	Business Annals (Abbreviated)
1899	101	99	−2	
1900	95	98	+3	Brief Recession
1901	99	101	+2	
1902	95	102	+7	
1903	93	100	+8	
1904	96	99	+3	Mild Depression
1905	106	103	−3	
1906	111	101	−9	
1907	105	99	−6	
1908	96	94	−2	Depression
1909	97	102	+5	
1910	95	104	+9	
1911	96	97	+1	Mild Depression
1912	103	99	−4	
1913	106	100	−6	
1914	98	99	+1	Depression
1915	110	99	−10	
1916	115	104	−10	
1917	104	103	−1	War
1918	98	107	+9	War
1919	102	111	+9	
1920	114	114	0	
1921	117	115	−2	Depression
1922	136	119	−13	

(1) Sum of deviations, without regard to sign = 125 per cent

(2) Average deviation = $\frac{125}{24}$ = 5.2 per cent

(3) Sum of deviations with regard to sign −68+57 = −11 per cent

(4) Average deviations with regard to sign = $\frac{-11}{24}$ = −.5 per cent

even if there were precise correspondence, it would not furnish any light upon the question as to whether capital for example should be privately owned to the degree to which it is in our society. For while capital may be "productive," it does not follow that the capitalist always is. Capital would still be "productive" even though its ownership were changed. Nor does it follow that the uses to which the capitalists put the income which they receive are on the whole socially the best. One may therefore be still a supporter of socialism, communism, or individualism and still square his social philosophy with the theory of production which we have developed.

10. *A Program for Further Work.*—In closing, it should be made clear that we do not claim to have actually solved the law of production,

but merely that we have made an approximation to it and suggested a method of attack. Future progress will be assisted by developing more refined series, by using different mathematical techniques, and by analyzing other sets of data.

Thus we may hope for: (1) An improved index of labor supply which will approximate more closely the relative actual number of hours worked not only by manual workers but also by clerical workers as well; (2) a better index of capital growth; (3) an improved index of production which will be based upon the admirable work of Dr. Thomas; (4) a more accurate index of the relative exchange value of a unit of manufactured goods.

In analyzing this data, we should (1) be prepared to devise formulas which will not necessarily be based upon constant relative "contributions" of each factor to the total product but which will allow for variations from year to year, and (2) will eliminate so far as possible the time element from the process.

We have developed our theory from the movement of labor, capital, production, value, and wages for the manufacturing industries of this country considered as a whole. There is opportunity to apply the same, or an improved method of analysis, to other lines of industry such as transportation, mining, public utilities, etc., in this country and to similar data for other countries. When this is done, we shall have most interesting material on the slope of the curves of imputed productivity for a wide variety of industries and may be able to frame combined curves for a country as a whole and from this frame interesting international comparisons.

Finally, we should ultimately look forward toward including the third factor of natural resources in our equations and of seeing to what degree this modifies our conclusions and what light it throws upon the laws of rent.

These are tasks which will require much time to complete but we submit that they are necessary if the precise relationships which probably lurk within economic phenomena are to be detected and measured.